Graph coloring
Exercise class problems - volume 1

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1. Prove that every graph has two vertices of the same degree.
2. Six people meet at a party. Prove that (1) there are three people that know each other or (2) there are three people that pairwise don't know each other.
3. In a class with 19 students each person sends a Valentine's Day card to exactly three other students. Is it possible that each student receives cards from the same three students to whom he/she sent cards?
4. Draw all isomorphism classes of trees on $n = 1, 2, 3, 4, 5$ vertices.
5. How many non-isomorphic trees on 10 vertices are there? (The answer is available online)
6. What is the maximal possible number of edges in a disconnected graph on n vertices?
7. Let G be a graph with minimum vertex degree $d \geq 2$. Prove that G contains a cycle of length at least $d + 1$.
8. Prove that if G is disconnected then its complement $\overline{G}$ is connected.
9. Classify all connected 2-regular graphs.
10. Prove that the k-cube Q_k is regular (of what degree)?
11. Find $\alpha(Q_k)$.
12. For which n is there a homomorphism $C_n \to K_2$?
13. For which n, m is there a homomorphism $C_n \to C_m$?
14. Construct a graph G such that the only homomorphism $G \to G$ is the identity.