Graph coloring
Exercise class problems - volume 2

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1. Prove that a graph with chromatic number k has at least $\binom{k}{2}$ edges.
2. Prove $\chi(G) \leq |V(G)| - \alpha(G) + 1$.
3. Suppose we have n straight lines in the plane, no three intersecting at the same point. Let G be the graph whose vertices are the intersection points of the lines, and two of them are adjacent if they appear consecutively on one of the lines. Prove that G is 3-colorable.
4. Suppose G is 4-colorable. Prove that G can be written as a union of two bipartite graphs (that means there are subgraphs H_1, H_2 of G such that $V(G) = V(H_1) \cup V(H_2)$, $E(G) = E(H_1) \cup E(H_2)$ and H_1, H_2 are bipartite).
5. Prove that if $\overline{G}$ is bipartite then $\chi(G) = \omega(G)$.
6. Prove that if an n-vertex graph G is k-colorable then G has at most $\frac{k-1}{2k}n^2$ edges.
7. Show that every graph has a vertex ordering for which the greedy algorithm uses exactly $\chi(G)$ colors.

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1. Write a function that takes a graph and an ordering of its vertices and produces a greedy vertex coloring for that ordering.
2. Write a function that does this for a 100 random orderings of the vertices and picks the most efficient coloring.