Graph coloring
Exercise class problems - volume 4

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1. What is $\min\{P_G(4) \ : \ G \text{ is planar}\}$?
2. Find the chromatic polynomial of
   - The $2 \times n$ grid $K_2 \square P_n$.
   - The wheel W_n.
   - $K_n - e$, the complete graph with one edge removed.
   - $P_n[K_m]$ (graph substitution, see Homework 1).
3. Let $P'_G(t)$ denote the number of colorings of G with *exactly* t colors $1, \ldots, t$ appearing, i.e. such that the color assignment $c : V(G) \to \{1, \ldots, t\}$ is surjective. Show that $P'_G(t)$ is not a polynomial for any graph G.
4. If G is a graph with chromatic polynomial $P(t)$, construct a graph with chromatic polynomial
   - $tP(t-1)$,
   - $(t-1)P(t)$,
   - $P(t)^3$,
   - $P(t)^2/t$.
5. Let G be a graph with n vertices, m edges and r triangles. Show that $[t^{n-2}]P_G(t) = \binom{m}{2} - r$.
6. Let G be a graph with n vertices, m edges and chromatic polynomial $P_G(t) = \sum_{i=0}^{n} a_i t^{n-i}$. Prove that $|a_i| \leq \binom{m}{i}$.