Graph coloring
Graph coloring game

Rules

We fix a graph G and a set of colors C.

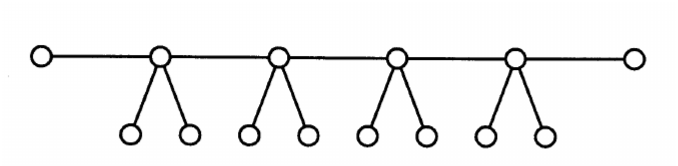
Two players, Alice and Bob, take turns coloring one vertex of G at a time, using a color from C, and so that any adjacent vertices which are already colored have different colors. Alice starts.

Alice wins when all vertices of G are colored (her aim is to color the whole graph). Bob wins if some player has no legal move (he is an adversary trying to stop Alice).

Some games to play

Who has a winning strategy?

- $G = P_3$, $C = \{1, 2\}$
- $G = P_6$, $C = \{1, 2\}$
- $G = C_6$, $C = \{1, 2\}$
- $G = C_8$, $C = \{1, 2, 3\}$
- $G = Q_3$, $C = \{1, 2, 3, 4\}$
- $G = Q_3$, $C = \{1, 2, 3\}$
- G is the tree below, and $C = \{1, 2, 3\}$



- Come up with your own pair (G, C), where Bob has a winning strategy, and you think this strategy is not easy to find.

Questions

Let $\chi_g(G)$ be the smallest number of colors k for which Alice has a winning strategy in the coloring game on the graph G with k available colors.

- Prove that $\chi(G) \leq \chi_g(G) \leq \Delta(G) + 1$.
- Find bipartite graphs with arbitrarily large χ_g.
- Describe all graphs with $\chi_g(G) \leq 2$.