

## Graph coloring

### Exercise class problems - volume 5

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Chromatic polynomials ctd.

1.  $P_G(t) = (t-1)^n + (-1)^n(t-1)$  implies that  $G$  is the  $n$ -cycle  $C_n$ .

Edge-chromatic numbers.

1. If  $G$  is connected and  $L(G)$  is isomorphic to  $G$  then  $G$  is a cycle.
2. Suppose that  $G$  has  $n$  vertices and  $m$  edges. Show that  $\chi'(G) \geq m/\lfloor n/2 \rfloor$ . Use it to show that the 5-vertex graph from the lecture has no 3-edge-coloring. Reprove that  $\chi'(K_{2k-1}) = 2k - 1$ .
3. We observed that  $\omega(L(G)) \geq \Delta(G)$ . Find an exact formula for  $\omega(L(G))$ .
4. Suppose  $G$  is  $r$ -regular with an odd number of vertices. Prove that  $r$  is even and that  $\chi'(G) = r + 1$ .
5. Find the edge-chromatic number of the Petersen graph.