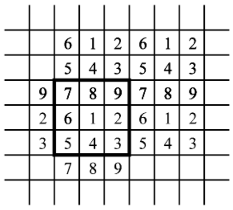
Graph coloring
Exercise class problems - volume 6

Edge-chromatic numbers, from last week:

1. If G is connected and $L(G)$ is isomorphic to G then G is a cycle.
2. Suppose G is r-regular with an odd number of vertices. Prove that r is even and that $\chi'(G) = r+1$.
3. Find the edge-chromatic number of the Petersen graph.

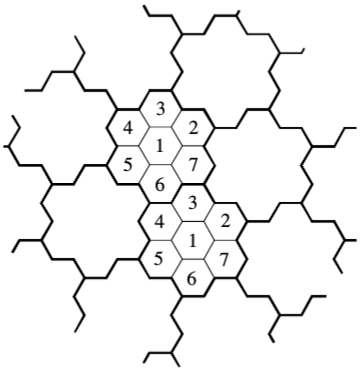
Coloring Euclidean spaces:

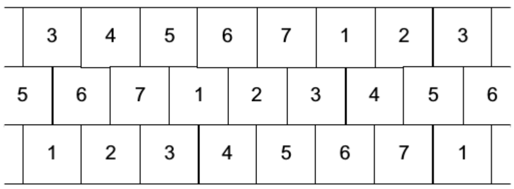
1. What is $\omega(U_{\mathbb{R}^2})$?
2. What is the length of the side in a d-dimensional cube whose main diagonal has length 1?
3. Recall our naive coloring of $U_{\mathbb{R}^2}$ by translates of a 9-colored 3×3 square, where each small square has diameter almost 1:

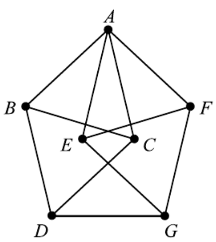


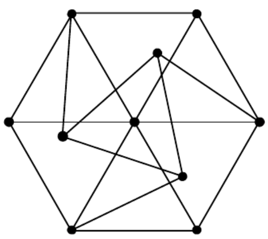
   Does the same strategy work in $\mathbb{R}^3$ and produce a coloring of $U_{\mathbb{R}^3}$ with 27 colors by translates of a 27-colored $3 \times 3 \times 3$ cube? What about $\mathbb{R}^d$?

4. These figures have all got something to do with the bounds $4 \leq \chi(\mathbb{R}^2) \leq 7$. Explain (or wait until the lecture tomorrow).









Bonus:

1. We say that two points A and B of the integer lattice $\mathbb{Z}^2$ *see each other* if the line segment AB contains no other point of $\mathbb{Z}^2$. Find a 4-coloring of $\mathbb{Z}^2$ so that any two points which see each other have different colors. Hint: use colors $00, 01, 10, 11$.