Graph coloring
Graded homework 3

Problem 1. (2pt) Find the edge-chromatic number χ' of `FunnyGraph(99)` from Homework 1.

Problem 2. ($2\frac{2}{3}$pt) In the lectures we defined a family of graphs $Q_d(u,s)$, and we used the fact that they are unit distance graphs in $\mathbb{R}^d$.

    a) Show that each $Q_d(u,s)$ is in fact a unit distance graph in $\mathbb{R}^{d-1}$.

    b) Use the graphs $Q_{10}(u,s)$ to prove $\chi(\mathbb{R}^9) \geq C$ for a constant C as large as you can.

Problem 3. (2.(6)pt) The *supremum metric* (or ℓ_∞ metric) in $\mathbb{R}^d$, $d \geq 1$ is given by

$$d_\infty((x_1,\ldots,x_d),(y_1,\ldots,y_d)) = \max\{|x_1 - y_1|,\ldots,|x_d - y_d|\}.$$

Find the smallest number of colors required to color $\mathbb{R}^d$ so that any two points whose distance in the supremum metric equals 1 have different colors.

Problem 4. ($\frac{8}{3}$pt) Let $P_{2\times n} = P_2 \square P_n$ be the $2 \times n$ grid graph.
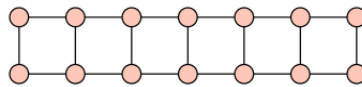
$$P_{2\times 7}$$

Find the number of edge-colorings of $P_{2\times n}$ with 3 colors. ($P_{2\times n}$ is called `graphs.Grid2dGraph(2,n)` in Sage).

Problem 5. (0pt) Which topics/theorems/methods from these lectures/exercises did you find most useful/useless/interesting/boring/easy/hard/... ?

Deadline: Friday exam week, 15/04/2016, 23:59.